Hedging and Regression

Returns

The discrete return on a stock is the percentage change: $\frac{S_i - S_{i-1}}{S_{i-1}}$.

The index i can represent days, weeks, hours etc.

What happens if we compute returns at infinitesimally short intervals of time?

$\frac{S_i - S_{i-1}}{S_{i-1}} = \frac{S_i}{S_{i-1}} - 1$ which is close to zero, and therefore it is very similar to

$$ln(S_i/S_{i-1})$$

.

Dow, daily prices.

Dow, weekly prices.

Dow, monthly prices.

Dow, monthly log(prices).

Returns

If I were to tell you that a certain stock XYZ went up $1 yesterday and ask you whether that is a lot what would you say?

Or, suppose that you are long that stock and they announced a stock split, does that affect you?

Returns

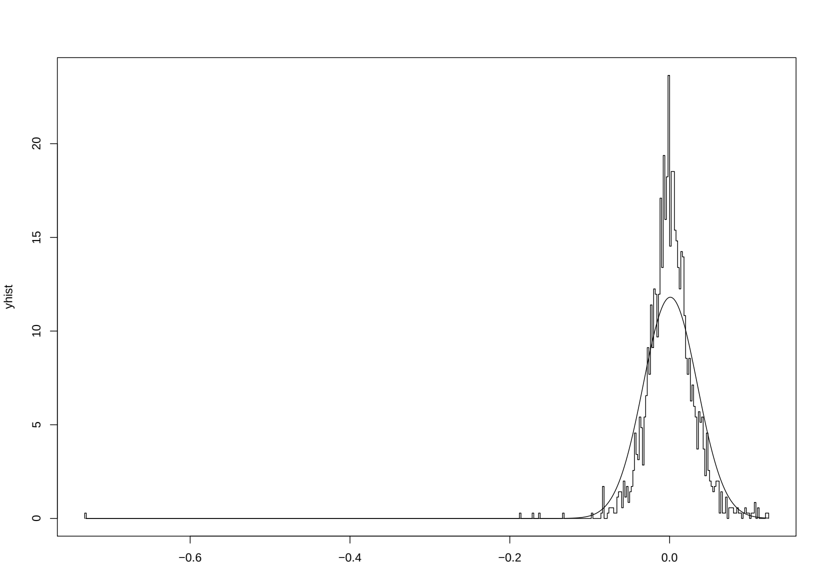
Prices are irrelevant..

The "real" variables we want to study are returns.

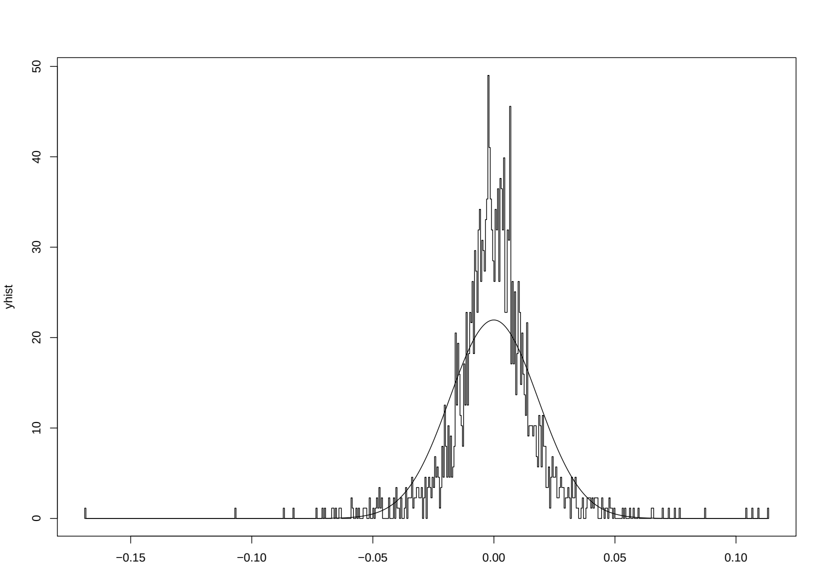
Last class we have seen some graphs of what prices look like.

Let's do the same with returns.

Apple Returns



IBM Returns



Returns

What is the distribution of the returns?

Or, better said, what model does it make sense to use for returns.

It looks like normality is a reasonable assumption..

Remind: A random variable is said to be normal with mean μ and variance σ^2 if

$$P(X \in A) = \int_A f(x)dx$$

where

$$f(x) = \frac{e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$$

It i denoted by $N(\mu, \sigma^2)$.

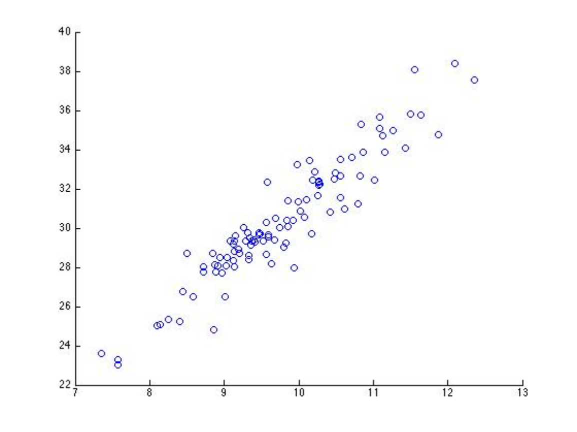
Review of Basic Regression

Suppose that we have a set of data pairs (x_i, y_i) and we want to model those linearly.

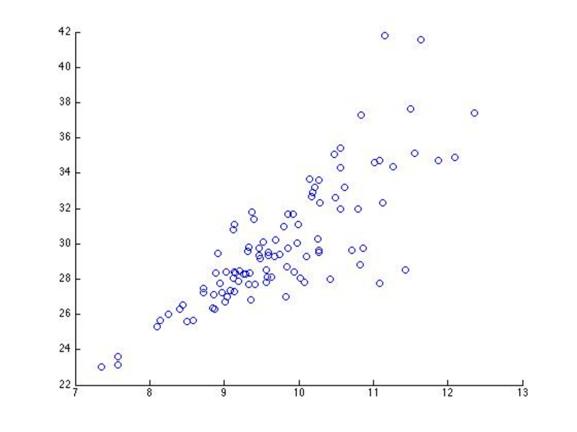
We propose the model $y_i = \alpha + \beta x_i + e_i$.

The errors e_i are assumed to be independent and $N(0, \sigma^2)$.

Review of Basic Regression



Review of Basic Regression



Review of Basic Regression

How do we find α and β?

Least squares:
We minimize $F(\alpha, \beta) = \sum_{i=1}^{n} |y_i - \alpha - \beta x_i|^2$

$$\frac{\partial F}{\partial \alpha} = -\sum_{i=1}^{n} 2(y_i - \alpha - \beta x_i) = 0$$

$$\frac{\partial F}{\partial \beta} = -\sum_{i=1}^{n} 2(y_i - \alpha - \beta x_i)x_i = 0$$

Review of Basic Regression

From there one gets that $\beta = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}$

If we define $s_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2$, $s_{yy} = \sum_{i=1}^{n}(y_i - \bar{y})^2$ and $s_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$ we can prove that:

$$\beta = \frac{s_{xy}}{s_{xx}}$$

On the other hand s_{xx}/n is an estimator for σ_x^2 and s_{xy}/n is an estimator of $\mathrm{Cov}(x, y)(= E(XY) - E(X)E(Y) = \rho\sigma_X\sigma_Y)$.

Then

$$\beta = \frac{\rho\sigma_y}{\sigma_x}$$

Relative Value

Suppose that you believe that Nokia will outperform the market in the near future. Based on this you decide to buy 1000 shares of Nokia and hedge it with Nasdaq futures (or with QQQ's).

The question is: how many Nasdaq futures do you have to sell?

We need to know how much Nokia moves when the Nasdaq moves by a point under "normal" circumstances.

We collect historical prices for both and run a regression of returns on returns (or, which is very similar, price changes on price changes).

The slope of the regression line gives the hedge ratio.

Relative Value

As we have seen, the slope of the line ends up being

$$\rho \frac{\sigma_{NOK}}{\sigma_{NDX}}$$

where:

ρ is the correlation coefficient between the returns of NOK and the returns of NDX.

σ_{NOK} is the standard deviation of the return of NOK (the volatility of Nokia).

σ_{NDX} is the standard deviation of the return of NDX (the volatility of the Nasdaq).

Notice that, provided that the correlation is not too bad, the hedge ratio is the ratio between the volatilities.

Hedging: Example 3.3

An airline expects to purchase 2MM gallons of jet fuel in 1 month and wants to hedge with heating oil futures.

Table 3.2 contains data on changes of heating oil futures and changes on jet fuel prices.

We use those values to compute $\rho = .928$, $\sigma_S = .0263$, $\sigma_F = .0313$ and then $\beta = \rho \frac{\sigma_S}{\sigma_F} = .78$.

Notice that $\sigma_F > \sigma_S$ means that the futures, on average, "move more" than the jet fuel prices.

Hedging: Example 3.3

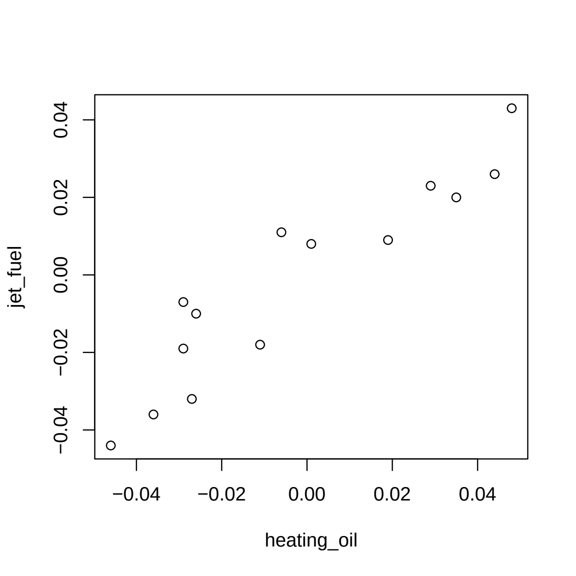
"Rough" interpretation of β: When F moves by 1, S moves by .78.

(It is not exactly like that since we have multiplied by the correlation coefficient)

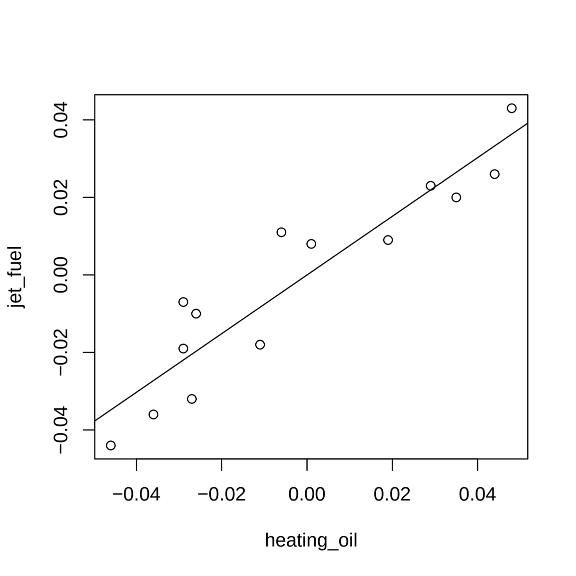
So, to hedge $2,000,000$ gallons of jet fuel we need less than $2,000,000$ gallons of heating oil.

How much less? $.78 * 2,000,000$.

Hedging: Example 3.3



Hedging: Example 3.3



Stack-and-Roll

Stack Hedge: Hedge the exposure with a single contract.

Stack-and-roll: Roll the hedge into the next contract when the one we are using expires.

Metallgesellschaft A.G.

In 1992, MG, a traditional metal company, had evolved into a provider of risk management services.

MGRM (US subsidiary of MG) committed to sell, at prices fixed in 1992, certain amounts of petroleum every month for up to 10 years.

MGRM sold forward contracts amounting to the equivalent of 160 million barrels.

Stack-and-Roll. Metallgesellschaft A.G.

MGRM employed a "stack-and-roll" hedging strategy.

It placed the entire hedge in short dated delivery months, rather than spreading this amount over many, longer-dated, delivery months.

In general people like to used short-dated contracts because of liquidity issues.

They got to be 16% of the open interest.

In September 1993 the market flipped from backwardation to contango.

In September 1993 they started to, consistently, lose money in their hedges.

In December the board decided to liquidate both the supply contracts and the futures positions used to hedge.

Related Links

- More on Metallgesellschaft (from IIT)
- Beta coefficient (from wikipedia)
- Market Neutral Long/Short Equity Trading (from Magnum Funds)