Interest Rate Swap

Financial institution pays 6$-$month Libor and receives 8% per anuum with semiannual compounding on $\$100,000,000$.

The swap has a remaining life of 1.25 years.

The Libor rates for 3$-$month, 9$-$month and 15$-$month are 10%, 10.5% and 11%. The 6$-$month Libor 3 months ago was 10.2%.

Interest Rate Swap

In terms of bonds:

Time	B_{fix}	B_{fl} *cf*	*Disc Factor*	*PV* B_{fix} *cf*	*PV* B_{fl} *cf*
.25	4.0	105.1	.9753	3.901	102.505
.75	4.0		.9243	3.697	
1.25	104.0		.8715	90.64	
Total				98.238	102.505

Note that the principal of the "implied" bonds are assumed to be the notional of the swap.

Interest Rate Swap

Then the value of the swap is:

$$V_{swap} = 98.238 - 102.505 = -4.267.$$

The fact that the value is negative means that should the institution want to get rid of the swap, sell it, it would have to pay for somebody to take it.

Interest Rate Swap

The same swap can be valued as a portfolio of FRAs.

We need to compute the 6−month forward starting in 3 months and in 9 months.
From

$$e^{.1*3/12} e^{f(0,3/12,9/12)*6/12} = e^{.105*9/12}$$

and

$$e^{.105*9/12} e^{f(0,9/12,15/12)*6/12} = e^{.11*15/12}$$

we get that

$$f(0, 3/12, 9/12) = .1075$$

and that

$$f(0, 9/12, 15/12) = .1175.$$

Interest Rate Swap

If we want to use the FRA formula that says

$$FRA\ Value = L(R_K - R_M)(T_2 - T_1)e^{-R_2 T_2}$$

we need to write 10.75% and 11.75% with semiannual compounding.

They are 11.044% and 12.102%.

Interest Rate Swap

Time	*Fixed cf*	*Floating cf*	*Net cf*	*df*	*PV*
.25	4.0	−5.1	−1.1	.9753	−1.073
.75	4.0	−5.522	−1.522	.9243	−1.407
1.25	4.0	−6.051	−2.051	.8715	−1.787
Total					−4.267

Currency Swap

A financial institution has entered into a currency swap paying 8% in dollars and receives 4% in yen once a year. The principals are \$10 million and $¥1,200$.

The swap has a remaining life of 3 years, suppose that the Libor curve is flat both in the US (9%) and in Japan (4%).

The current exchange rate is $¥110 = \$1$.

Again, we can value as bonds or as forward contracts.

Currency Swap

In term of bonds:

Time	*Cf \$ bond*	*PV \$*	*Cf ¥ bond*	*PV ¥*
1	.8	.7311	60	57.65
2	.8	.6682	60	55.39
3	.8	.6107	60	53.22
3	10.0	7.6338	1,200	1,064.3
Total		9.6439		1,230.55

Currency Swap

Then $B_{\$} = 9.6439$ and $B_{¥} = 1,230.55$.

Since the exchange rate is ¥110 = \$1 :

$$V_{swap} = \frac{1,230.55}{110} - 9.6439 = 1.5430 \text{ million}$$

Currency Swap

As forward rates:

Knowing both interest rates curves (both flat) and the current exchange rate allows us to compute the forward exchange rate.

For example the $1-$year rate is:

$$F(0,1) = 110e^{(.04-.09)*1} = 104.6352 \text{ ¥ per \$}$$

or

$$\frac{1}{F(0,1)} = \frac{1}{104.6352} = .009557 \text{ \$ per ¥}$$

Currency Swap

Then, 60 million ¥ in 1 year are worth $60 * .009557 = .5734$ in "1−year" dollars.

So, the net cash flow at the end of 1 year is $0.8 - 0.5734 = -.2266$ in million $.

Therefore, to discount that to the present I have to use the dollar interest rate.

I could have computed the cash flow in ¥ but, in that case, I would have used the japanese rate.

Currency Swap

Time	*Cf* $	*CF* ¥	*Fwdrate*	$ *value of* ¥ *cf*	*Net cf in* $	*PV*
1	−.8	60	.009557	.5734	−.2266	−.2071
2	−.8	60	.010047	.6028	−.1972	−.1647
3	−.8	60	.010562	.6337	−.1663	−.1269
3	−10.0	1,200	.010562	.12.6746	2.6746	2.0417
Total						1.543

So, both approaches agree in the value..