FM 5022
Hw9

1) Suppose that we define the instantaneous forward rate for time s (observed at time $t = 0$) as:

$$f(0, s) = \lim_{s_2 \to s^+} f(0, s, s_2)$$

Prove that:

$$e^{-\int_0^s f(0,s)ds} = \hat{E}(e^{-\int_0^s r(s)ds})$$

2) Suppose that we consider the money market account with two compounding periods.

Let us denote $t_1 - t_0$ and $t_2 - t_1$ the two time intervals and r_1, r_2 the interest rates on each time interval.

These interests are stochastic, they are only known at the beginning of the period.

a) Consider going long $e^{r_1(t_1-t_0)}$ futures at time t_0. What is the cash-flow at time t_1?

b) At time t_1 increase your long position, take a total position of $e^{r_1(t_1-t_0)}e^{r_2(t_2-t_1)}$

c) What is the final cash-flow?

d) Compute $\hat{E}(e^{-r_1(t_1-t_0)-r_2(t_2-t_1)}\text{payoff}_2)$

e) The futures price is the value of K that for which that expected value is 0.

f) Conclude that the futures price is $\hat{E}(S_T)$.

3) A forward contract for an asset S_t expiring at time T pays $S_T - F(0, T)$. Use this to prove that the forward price of S at time 0 ($F(0, T)$) is the expected value of S_T in the world which is forward risk-neutral with respect to $P(t, T)$.