Math 5022
Midterm

Name:

1) Consider a portfolio containing four stocks S_1, S_2, S_3 and S_4. Their (annualized) volatilities are .8, .25, .45 and .15 respectively. Suppose, also, that the correlations are $\rho_{1,2} = .6$, $\rho_{3,4} = .4$, $\rho_{1,3} = \rho_{1,4} = \rho_{2,3} = \rho_{2,4} = 0$.

a) Find the eigenvectors and eigenvalues of the correlation matrix.

b) If the portfolio consists of 3MM worth of S_1 and 1MM of (each) S_2,S_3 and S_4. What is the $1-$day 99% VAR?

2) Suppose that X is a continuous random variable that follows an exponential distribution with parameter $\lambda > 0$. This means that the density of X is:

$$f(x) = \lambda e^{-\lambda x}, x \geq 0$$

a) Compute $E(X)$ and $V(X)$.

b) If $x_1, \ldots, x_n$ is a sample corresponding to one such X find the maximum likelihood estimator of the parameter λ.

3) A certain time series is modeled with the AR(1) process:

$$x_t = 8.1 + .8x_{t-1} + \epsilon_t$$

where ϵ_t is a white noise with $E(\epsilon_t^2) = .5$.

a) Is x_t a stationary process? Why? Prove it.

b) What are the mean and the variance of x_{100}?

c) Consider another AR(1) process z_t which follows a similar equation:

$$z_t = 8.1 + z_{t-1} + \epsilon_t$$

How does your answer to question (a) change? Justify.

4) Suppose that you need to price a call on a spread of two stocks S^1 and S^2. You know that:

$$S_0^1 = 100, S_0^2 = 90, r = 0, q^1 = 0, q^2 = 0, T = 4 \text{ months } .\sigma^1 = .2, \sigma^2 = .3, K = 10$$

In addition, you know that the correlation between the returns of S^1 and the returns of S^2 is .8.

You decide to price the option using Monte Carlo simulation with $\Delta t = 1 \text{ month}$ and then, to generate the first path, you draw 8 (normal) numbers from a random number generator $.01, 1.15, -.2, .51, -.6, .04, -1.01, .43$.

a) How do you generate a path of the spread $S^1 - S^2$ using those 10 numbers? (S^2 denotes the second stock, it does not mean a square).

b) What will the payoff be for that path?

c) Is $S_T^1 - S_T^2$ a lognormal variable? Explain.

5) As we know the Black-Scholes formula for a call option is given by: $Se^{-qT}N(d_1)-Ke^{-rT}N(d_2)$ where $d_1 = \frac{\log(S/K)+(r-q+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$ and $d_2 = d_1-\sigma\sqrt{T}$. Using this formula and put-call parity deduce the Black-Scholes formula for a put on S with the same strike K.