Math 5021
Final

1) Suppose that S follows the $\frac{dS}{S} = \mu_1\, dt + \sigma_1 dW$ during 2020 and $\frac{dS}{S} = \mu_2 dt + \sigma_2 dW$ during 2021. Assume that today is $1/1/2020$.

a) What is the distribution of $S_{.5}$? (time in years, so .5 means after half a year)

b) What is the distribution of S_2?

c) What is the SDE followed by S^5? You can used fixed μ and σ for this one.

2) On January 1, 2020 the treasurer of a certain company learns that she will need to borrow $1,000,000 on January 1, 2021 for a year. The treasurer calls a bank and asks how much she would have to agree to pay in interest if the transaction is done today.

a) Use the following yield curve (given in continuously compounded convention) to find out what the answer from the bank should be:

Maturity (Yrs)	Rate (annualized)
.5	1.0 %
1.0	1.5 %
1.5	1.5 %
2.0	2.0 %
2.5	2.5 %

b) It turns out that, after 6 months, the treasurer realizes that she will not need to borrow money. Given that the the yield curve now looks like this:

Maturity (Yrs)	Rate (annualized)
.5	.5 %
1.0	1.0 %
1.5	1.0 %
2.0	2.0 %
2.5	3.0 %

Did she make or lose money in the trade? How much?

3) Suppose that on 1/1/2020 you are long a swap starting in 1 year and expiring in two years which pays 1% fixed. So, it consists of two periods, first payment in 1.5 years and second payment in 2 years. By being long you get paid the fixed leg and pay the floating leg. Assuming the same curve as in the previous problem, what is the value of this swap?

4) Suppose that stock XYZ is trading at \$100. An investor is considering buying an atm call which is trading at 20% volatility, giving a price of \$6.1. According to a vol forecasting model used by the investor this price appears to be expensive. Therefore, the investor decides to sell the call and hedge the directionality of the position. Assuming that the investor can trade fractional amounts of stock and that:

$$\Delta = .5$$

and

$$\Gamma = .03$$

a) Describe the portfolio to be held by the investor.

b) If right after inception XYZ trades at \$100.5, how would you estimate the new value of this portfolio?

c) With XYZ trading at \$100.5 the portfolio is not delta neutral. What trades should the investor do to rebalance the hedge?

5) A futures contract is trading at 100.

3 a) What is the price of a 6-month european $80 put ($K = 80$) if the risk-free rate is 1.5% and the volatility is 20%?

b) What are the delta and gamma of the option?

c) Compute the price of the same put but in its american version.