FM 5022
Hw11

1) Black, Derman and Toy proposed the following model for r:

$$d\ln(r) = (\theta(t) + \frac{\sigma'(t)}{\sigma(t)}\ln(r))dt + \sigma(t)dw$$

Solve the equation for r (i.e. find an integrating factor and integrate the resulting equation).

2) Suppose that X and Y follow Itô processes with diffusion part: $dX = \sigma_{1,1}dW_1 + \sigma_{1,2}dW_2$ and $dY = \sigma_{2,1}dW_1 + \sigma_{2,2}dW_2$ where dW_1, dW_2 are independent. This means that the normal variables $W_1(t+h) - W_1(t)$ and $W_2(t+h) - W_2(t)$ are independent.

a) What is the expected value of dX ? (Remember $dX \approx X(t+h) - X(t)$)
b) What is the variance of dX?
c) What is the covariance of dX and dY ?
d) What is the correlation of dX and dY ?
e) Construct an example (choose the σ's) in which dX and dY are uncorrelated and one in which they are perfectly correlated.

3) Suppose that a process y follows the stochastic differential equation:

$$dy = (2\alpha y + \sigma^2)dt + 2\sigma\sqrt{y}\, dW$$

a) What is the sde followed by $z = \sqrt{y}$?

b) Confirm that you got the right thing by using the sde that you got for z to get back to dy (by setting $y = z^2$).
(So, r in the CIR model is just the square of a variable which mean reverts around 0)

4) From the book, solve: 31.4.