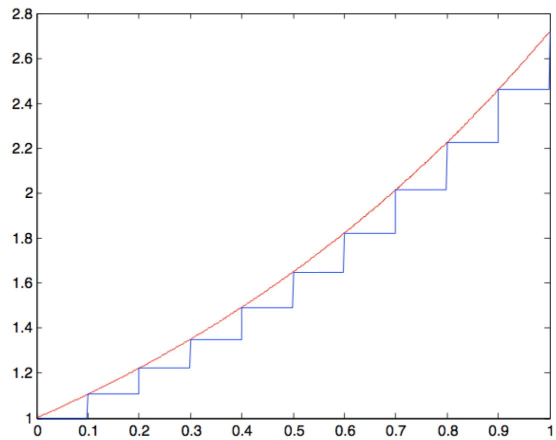
FM 5022
Hw 10

1) We want to figure out what the variance of $\int_0^t e^{\alpha s} dW(s)$ is.

To do this we can proceed in a number of ways. One of these is to go straight from the definition of the stochastic integral.

a) Let us consider a partition of the interval $[0, t]$ in n subintervals of length $\frac{t}{n}$ and define the processes $X_s^{(n)} = e^{\alpha \frac{k}{n} t}$ if $\frac{k}{n} t \leq s < \frac{k+1}{n} t$ where $k = 0, .., n-1$.



What is $\int_0^t X_s^{(n)} dW(s)$?
b) What is $E(\int_0^t (X_s^{(n)})^2 ds)$?
c) Is it true that

$$\lim_{n \to \infty} E(\int_0^T (e^{\alpha s} - X_s^{(n)})^2 ds) = 0?$$

Why? (Notice that the expected value here is not necessary since the integrand is deterministic.
d) Conclusion: what is the variance of $\int_0^t e^{\alpha s} dW(s)$?

2) Remember that Vasicek's model specifies the short rate r as

$$dr = a(b - r)dt + \sigma dW$$

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a) Solve the equation.
b) Prove that the deterministic part of $r(T)$ is a weighted average of $r(0)$ and b.

c) Let $\delta t = t_i - t_{i-1}$. Apply your solution to the equation from time t_{i-1} to time t_i. Assuming that you know $r_{t_{i-1}}$, use the solution you found in part (a) to write an expression for r_{t_i}.

d) Argue that the equation that you found is an autoregressive process of order 1 ($AR(1)$).

e) Is it stationary?

In addition, solve problems 29.3 and 29.5 from the book.