# Valuation of bonds paying floating coupons

# Floating coupon paying bonds

How much would you pay for a bond expiring in three months and paying 100 plus a coupon equal to todays three-month rate??

$$B = \frac{100 + c100}{(1 + r/4)}$$

what is the relationship between $r$ and $c$?

$c = r/4$, therefore $B = 100$.

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What about a bond that expiries in six months and at the beginning of each three-month period it sets its coupon according to the 3−month rate?
I know the first coupon $c_1 100$ just as before.
However, I do not know what the second coupon will be.
Still, I can lock in $f(0, t_1, t_2)$ by entering into an FRA.
I claim that the price of the bond should be

$$B = \frac{100r/4}{(1 + r/4)} + \frac{f(0, t_1, t_2)100 + 100}{(1 + r/4)(1 + f(0, t_1, t_2))}$$

$$B = \frac{100(1 + f(0, t_1, t_2))r/4 + 100(1 + f(0, t_1, t_2))}{(1 + r/4)(1 + f(0, t_1, t_2))} = 100$$

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Why should the claim be true?

We can hedge every cash-flow today, locking-in the forward rates indicated by the yield curve today.

In other words, for the price of 0 we can make the bond into a bond paying the implied forward rates today at each coupon day.

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How do I prove the claim?

If $B > 100$:
1) Sell the bond, invest the proceeds for six months at $R(0, .5)$ and lock-in $f(0, .25, .5)$ for the second three months for a notional of $100R(0, 25)$. Also buy an FRA for \$100 for the interval $[.25, .5]$, $R_K = f(0, .25, .5)$.

2) In three months borrow $100R(0.25)$ for three months at $f(0, .25, .5)$ and pay the coupon.

3) In six months pay the coupon $100R(.25, .5)$ and get from the FRA $100(R(.25, .5) - f(0, .25, .5))$, also pay \$100 and pay the loan $100R(0, .25)(1 + f(0, .25, .5))$

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How do I prove the claim?

4) In total we payed
$100 + 100R(0,.25)(1 + f(0,.25,.5)) + 100f(0,.25,.5) =$
$100(1 + R(0,.25)(1 + f(0,.25,.5))$

5) Form investing $B$ we get $B(1 + R(0,.25))(1 + f(0,.25,.5))$.

6) Gain: $(B - 100)(1 + R(0,.25))(1 + f(0,.25,.5))$ which is exactly $B - 100$ at time 0.

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What is the price on any other date? (not a coupon-paying date).

Suppose that the bond pays coupons semianually and it has paid a coupon 2 months ago. Then, 32 months ago, it has also fixed the coupon for the next period.

So right now we know what the coupon will be in 4 months. Let us assume that the coupon paid in 4 months will be 8% ($4).

In 4 months, the bond will pay $4 and right after that the bond will be worth $100.

So the price of the bond right *before* the payment of the coupon will be $104.

So, the price today has to be $104e^{-R(0,4/12)*4/12}$,
where $R(0,4/12)$ is the 4−month Libor today.