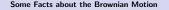
Some Facts about the Brownian Motion

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•
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