Some Facts about the Brownian Motion

Remember: a stochastic process X is said to be a martingale if your best guess for the future is the current value.

In other words, $\mathbb{E}(X_t|\mathcal{F}_s) = X_s, s \leq t$.

Sometimes you will see written $\mathbb{E}(X_t|X_s)$ instead of $\mathbb{E}(X_t|\mathcal{F}_s)$.

Remember: a stochastic process X is said to be a martingale if your best guess for the future is the current value.

In other words, $\mathbb{E}(X_t|\mathcal{F}_s) = X_s, s \leq t$.

Sometimes you will see written $\mathbb{E}(X_t|X_s)$ instead of $\mathbb{E}(X_t|\mathcal{F}_s)$.
Let W_t be a Brownian Motion,

- W_t is a martingale.

Remember: a stochastic process X is said to be a martingale if your best guess for the future is the current value.

In other words, $\mathbb{E}(X_t|\mathcal{F}_s) = X_s, s \leq t$.

Sometimes you will see written $\mathbb{E}(X_t|X_s)$ instead of $\mathbb{E}(X_t|\mathcal{F}_s)$.
Let W_t be a Brownian Motion,

- W_t is a martingale.
  Notice that, as $W_0 = 0$ this implies that for any $t \geq 0$, $\mathbb{E}(W_t) = 0$.

Remember: a stochastic process X is said to be a martingale if your best guess for the future is the current value.

In other words, $\mathbb{E}(X_t|\mathcal{F}_s) = X_s, s \leq t$.

Sometimes you will see written $\mathbb{E}(X_t|X_s)$ instead of $\mathbb{E}(X_t|\mathcal{F}_s)$.
Let W_t be a Brownian Motion,

- W_t is a martingale.
  Notice that, as $W_0 = 0$ this implies that for any $t \geq 0$, $\mathbb{E}(W_t) = 0$.

  Now $W_t^2 \geq 0$ and sometimes is strictly bigger, therefore, it can't be a martingale.

Remember: a stochastic process X is said to be a martingale if your best guess for the future is the current value.

In other words, $\mathbb{E}(X_t|\mathcal{F}_s) = X_s, s \leq t$.

Sometimes you will see written $\mathbb{E}(X_t|X_s)$ instead of $\mathbb{E}(X_t|\mathcal{F}_s)$.
Let W_t be a Brownian Motion,

- W_t is a martingale.
  Notice that, as $W_0 = 0$ this implies that for any $t \geq 0$, $\mathbb{E}(W_t) = 0$.

  Now $W_t^2 \geq 0$ and sometimes is strictly bigger, therefore, it can't be a martingale.
  However:

Remember: a stochastic process X is said to be a martingale if your best guess for the future is the current value.

In other words, $\mathbb{E}(X_t|\mathcal{F}_s) = X_s, s \leq t$.

Sometimes you will see written $\mathbb{E}(X_t|X_s)$ instead of $\mathbb{E}(X_t|\mathcal{F}_s)$.
Let W_t be a Brownian Motion,

- W_t is a martingale.
  Notice that, as $W_0 = 0$ this implies that for any $t \geq 0$, $\mathbb{E}(W_t) = 0$.

  Now $W_t^2 \geq 0$ and sometimes is strictly bigger, therefore, it can't be a martingale.
  However:
- $W_t^2 - t$ is a martingale.